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A STUDY ON THE EIGEN-MODE INFLUENCED BY THE INTERFACE BETWEEN TWO DIFFERENT DIELECTRICS

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ABSTRACT

FEM has been carried out to investigate the eigen-mode of a square cross section waveguide filled with two different dielectrics. Vector Helmholtz equation was re-configured as transverse space dependent form to facilitate FEM. The eigen-equation of FEM was established from element equations of triangular mesh. For the transverse vectot field, the equation was constructed using tangential edges of the element mesh. The equation for the transverse vector potential component was made basing on its nodal points. These equations were combined into a gloval square matrix to calculate the eigen-modes. TE and TM eigen-modes influenced by the interface were obtained using the Krylov-Schur iteration method. As a result, each transverse vector fields and their vector potential component have been revealed by schematic representation to identify their physical property. And spectra of TM eigen-mode were compared with results of the theoretical calculation.

KEYWORDS: FEM, eigen-mode, Helmholtz euation, Krylov-Schur iteration method, TE and TM mode.

INTRODUCTION

It has been well known that the interface between two different dielectrics dominantly characterize the eigen-property of the waveguide[1]. By Snell's law, the beam cannot be propagated through the interface of different dielectrics when its incident angle is greater than the critical angle. It is also known that partially remained evanescent beam from the incident beam would be going along the interface between dielectrics. Although it would disappear shortly after producing, understanding about their eigen-property may give an important physical meaning to various fields[2]. However, it may not be easy matter to understand these property even though the interface made with simple geometrical structure. Theoretical attempts to understand the eigen-property have not yet provided any satisfying results coincide with reality.

As an alternative to those difficulties, numerical analysis may be utilized for a proper understanding of the optical properties of the various interfaces. Among them, it has been well known that FEM(Finite element method) is the most prominent numerical analysis method[3]. In this study, we investigated the influence of the interface on the eigen-modes forming in the waveguide by FEM. A cross section of the waveguide is constructed with two different dielectrics. The interface between them is formed along the breadthwise direction. It is assumed that the waveguide has constant dielectrics along the longitudinal direction. In the course of FEM analysis, the cross section was divided into triangular mesh. The eigen-matrix equation was constructed using tangential edges and nodes of triangular element meshes. Tangential edges and nodes were used as calculating variables for transverse vector fields and their vector potential component respectively.

The eigen-modes were obtained by using the Krylov-Schur iteration method[4]. TE(Transverse Electric) and TM(Transverse magnetic) modes could be calculated by giving different boundary conditions for them. Each columns of the similarity transforming matrix contain the eigen-mode of transverse vector field and its vector potential component simultaneously. Among the eigen-modes, several prominent TE and TM vector fields accompanying with their vector potential component were revealed by schematic presentation. To understanding the characteristics of their eigen-property, these spectra were compared with results of homogeneous waveguide. And in order to confirm the calculation reliability, these spectra were also compared with results from the theoretical calculation.

FINITE ELEMENT FORMULATION

The characteristic electromagnetic wave of the waveguide depend on the spacial distribution of the dielectric materials. In this study, it was assumed that the waveguide are constructed with two different dielectrics as fig.1.

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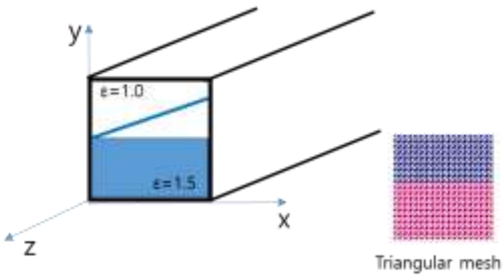

Fig. 1 The Cross Section of the Waveguide

The vector Helmholtz equation would be used in describing the wave property in the waveguide of square cross section. It is described as following equation[5]

$$\vec{\nabla} \times (\boldsymbol{p}\nabla \times \vec{\boldsymbol{F}}) - \boldsymbol{k_o^2 q}\vec{\boldsymbol{F}} = \mathbf{0} \qquad (1)$$

Where k_o is the wave number and, for $\vec{F} = \vec{E}$(electric field strength), p $= 1/\mu_r(\mu_r$: relative permeability $\mu/\mu_o)$, q $= \varepsilon_r(\varepsilon_r$: relative permittivity $\varepsilon/\varepsilon_o$) and, for $\vec{F} = \vec{H}$ (magnetic field strength), p $= 1/\varepsilon_r$, q $= \mu_r$. For convenience of calculation, the common notations $\vec{F}$, p and q would be used to relate these values without differentiating electromagnetic field modes. The eigen-equation has been established from FEM. The shape functions for the triangular element mesh are calculated as following

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}^{-1} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} = \frac{1}{2A} \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \qquad (2)$$

where $a_i = x_j y_k - x_k y_j, b_i = y_j - y_k, c_i = x_k - x_j$ (i, j and k are cyclical ordering) and A is the area of the triangular element mesh. Relating with these shape functions, the constant tangential edge vectors for the element mesh are given by[6]

$$\vec{W_m} = L_m(N_i \vec{\nabla}_t N_j - N_j \vec{\nabla}_t N_i) \qquad (3)$$

where $\vec{\nabla}_t = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}$, $L_m$is the length connecting nodes i and j. It is more convenient to express these vectors by using the shape function coordinates.

$$\vec{W_m} = \frac{L_m}{4A^2}[(A_m + B_m y)\hat{x} + (C_m + B_m x)\hat{y}] \qquad (4)$$

where $A_m = a_i b_j - a_j b_i$

$B_m = c_i b_j - c_j b_i$

$C_m = a_i c_j - a_j b_i$

$D_m = b_i c_j - b_j c_i = -B_m$

With these tangential edge vectors and shape functions, the transverse vector fields and their vector potential component of each element mesh can be written as

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = \begin{bmatrix} \{W_x\}^T & 0 \\ \{W_y\}^T & 0 \\ 0 & j\{N\}^T \end{bmatrix} \begin{bmatrix} \{F_t^e\} \\ \{F_z^e\} \end{bmatrix} \qquad (5)$$

The Galerkin method of weighted residual has been used to construct the eigen-equation. The envelope function concept for the element vector

$$\{F\} = \begin{Bmatrix} \vec{F}_t \\ F_z{}' \end{Bmatrix} \exp(-j\beta)$$

was used where β is a propagation constant. The eigen-equation was obtained from the (0,1) Pade approximation of propagation scheme at the beginning position[7]

$$[A]\{F\} = -\frac{1}{2k_o n\beta}[B]\{F\} \qquad (7)$$

where $[A] = \begin{bmatrix} [G] & [E] \\ [F] & [D] - k_o^2[I] \end{bmatrix}$

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$$[B] = \begin{bmatrix} k_o^2 n_o^2 [G] - k_o^2 [H] + [C] & k_o^2 n_o^2 [E] \\ k_o^2 n_o^2 [F] & k_o^2 n_o^2 ([D] - k_o^2 [I]) \end{bmatrix}$$

and for the calculational stability $F_z' = j \frac{\partial}{\partial z} F_z$ was adopted. The matrices components were calculated as following

$$[C] = \iint_A p(\vec{\nabla}_t \times \vec{W}_m) \cdot (\vec{\nabla}_t \times \vec{W}_n) ds = p \frac{L_m L_n}{4A^3} D_m D_n$$

$$[\mathrm{D}] = \iint_A p(\vec{\nabla} N_i) \cdot (\vec{\nabla} N_j) ds = \mathrm{p} \frac{b_i b_j + c_i c_j}{4A}$$

$$[\mathrm{E}] = \iint_A p \vec{W}_m \cdot \vec{\nabla} N_j ds = p \frac{L_m}{8A^2} [b_j (A_m + B_m \bar{y}_{tri}) + c_j (C_m + D_m \bar{x}_{tri})$$

$$[\mathrm{F}] = \iint_A p \vec{\nabla} N_i \cdot \vec{W}_n ds = p \frac{L_n}{8A^2} [b_i (A_n + B_n \bar{y}_{tri}) + c_i (C_n + D_n \bar{x}_{tri})$$

$$[G] = \iint_A p \vec{W}_m \cdot \vec{W}_n ds = p \frac{L_m L_n}{16A^3} \sum_{a=1}^{a=5} I_a$$

where $I_1 = (A_m A_n + C_m C_n)$\`

$$I_2 = (C_m D_n + D_m C_n) \bar{x}_{tri}$$
$$I_3 = (A_m B_n + B_m A_n) \bar{y}_{tri}$$
$$I_4 = \frac{B_m B_n}{12} (\sum_{i=1}^{i=3} y_i^2 - 9 \bar{y}_{tri}^2)$$
$$I_5 = \frac{D_m D_n}{12} (\sum_{i=1}^{i=3} x_i^2 - 9 \bar{x}_{tri}^2)$$

$$[H] = \iint_A q \vec{W}_m \cdot \vec{W}_n ds = q \frac{L_m L_n}{16A^3} \sum_{a=1}^{a=5} I_a$$

$$[\mathrm{I}] = \iint_A N_i N_j ds = q \frac{A}{12} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

In these representations, subscripts for N and $\vec{W}$ indicate the node and edge numbers respectively. These element matrices have been assembled over all triangular elements to obtain a global eigen-matrix equation.

As mentioned in the previous study, it has been well known that the Krylov-Schur iteration method is the most reliable technique for finding the prominent eigen-modes[8]. The method would be more efficiently implemented in finding specific eigen-pairs by performing the shift-invert strategy as following[9]

$$\lambda_o \{F\} = \frac{[B]}{[A] - \sigma[B]} \{F\} = [M]\{F\} \qquad (8)$$

where $\lambda_o = \frac{1}{-\frac{1}{2nk_o\beta} - \sigma}$. The sparsity and symmetry of the eigen-equation would be lost, but by this strategy the convergent rate is more promoted at the specific value σ. Subsequently, the Krylov-Schur iteration method is performed on this square matrix [M].

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RESULTS AND DISCUSSION

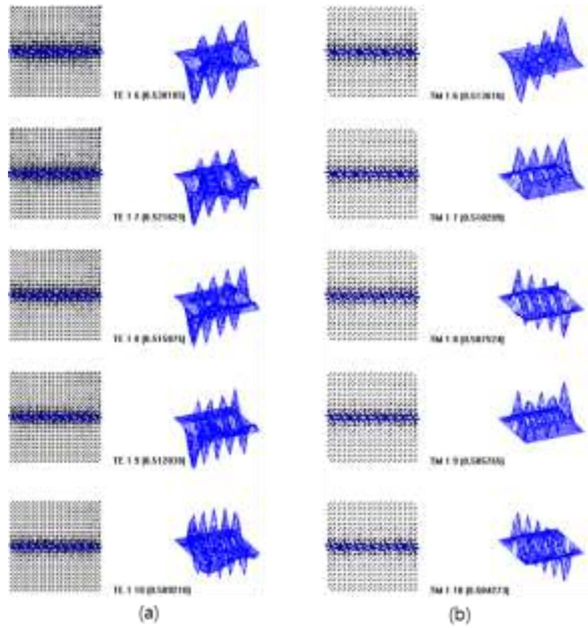

Fig. 2 Eigen-modes and Propapation Numbers

In this study, the eigen-modes influenced by the interface of two dielectrics were investigated. As can be seen from fig.1, the waveguide consisted of two dielectrics having different dielectric constant. The lateral surface of the waveguide was assumed to be a perfect conductor. In calculating process, this assumption provides convenience to obtain TM modes by ignoring the variables on the surface.

As a result, spectra are revealed schematically in fig.2. These results were obtained by performing the Krylov-Schur iteration on the square matrix $[M]$ of eq. 8. The eigen-modes were the column vectors of the similar transforming matrix which convert the square matrix to a Shure form. The eigen-values were calculated by converting each diagonal component of the Schur matrix into value $-\frac{1}{2nk_o\beta} = \frac{1}{\lambda_o} + \sigma$ reversing the shift-invert strategy. The propagation constants calculated from this relation were written in the blanket under each spectrum. The reflectivity n was assumed to be an average value of the waveguide. The wave number was set to be $k_o = 1$ for convenience. The eigen-modes have been represented with two components. The one is for the transverse electromagnetic vector field. The other is vertical component of vector potential of waveguide. The former was resulted from the tangential edge vectors and the latter was obtained from the nodes of the triangular element. The left columns of fig. 2(a), (b), represents transverse modes which have been obtained by applying eq. 5 to corresponding tangential edge variables of the triangular elements. The right columns of fig. 2(a), (b), represent components of their corresponding vector pontential resulted by applying eq. 5 to their node variables. The spectra of fig.2(a) are TE modes which represent the standing wave formed in the waveguide of the opened lateral boundary. As can be seen from these spectra, the peak position occurs along to the interface. The spectra of fig.2(b) are TM modes which represent the standing wave formed in the waveguide of closed lateral surface. From these representations, it can be identified that their maximum amplitudes occur at the minimum points of TE modes.

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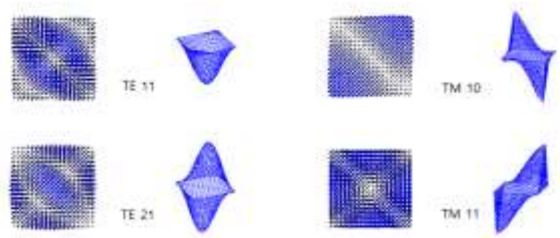

Figure 3Eigen-modes in the homogeneous waveguide

It has been identified that, eigen-modes are developed especially at the interface of two dielectrics. Unlike homogeneous waveguide, the eigen-modes in this system tend to reflect the characteristics of the interface. As a comparison, several prominent eigen-modes formed in a homogeneous medium are illustrated in fig. 3. In general, the eigen mode is determined dominantly by boundary conditions of the corner, surface and the interface etc.. The spectra in fig.3 suggest that the corner or surface may have affected the eigen-modes. As can be seen in fig. 2, the eigen-modes are formed around the interface tightly. By comparing these spectra, it has been identified that there are clear differences between them. So, it has been confilmed that the eigen-modes in this study relies on the interface dominantly. In reality, finding a specific eigen-mode has been depended entirely on the specific value σ of the shift-invert strategy eq.8. This value has been selected through a lot of trials and errors. Finally, these values have been determined to be –3.955 and –400 for TE and TM modes respectively.

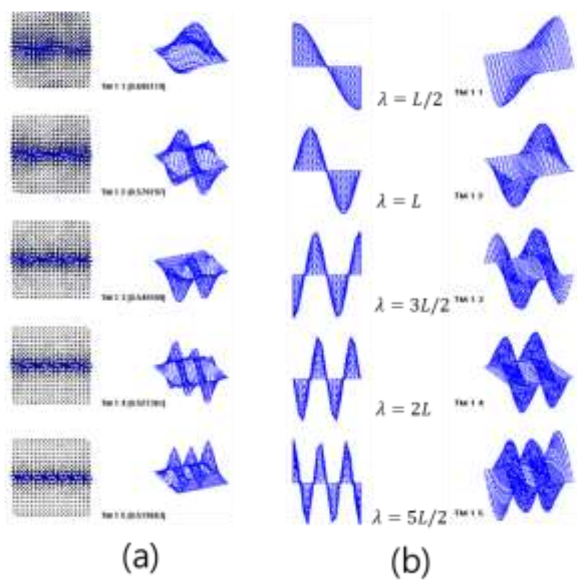

Fig. 4 FEM and theoretical spectra of TM modes

Generally, to understand the properties of propagating waves in the waveguide, it would be demanded to understand a stable eigen-mode. Among the eigen-modes obtained in this study, TM modes revealed the more stable and prominent characteristics at low eigen-number than TE modes. TM modes of the low eigen-number are shown in fig.4. Fig.4(a) is schematic representation of TM modes extending to fig.2(b). Fig.4 (b) is the spectra basing on the theoretical calculation of C. A Balanis[10]. These spectra were obtained using the following vector potential function

$$F_y \sim A\cos(\frac{m\pi}{L}x)\sin(\frac{n\pi}{L}(b-y)) \qquad (9)$$

where parameters m, n are the mode numbers of the spectra. b is the position of the interface from the lower surface of the waveguide. And L is the width of the waveguide. At first glance, the schematic representations for the potential function of TM 1 3 and TM 1 5 seem not to be the same with FEM. But these spectra can be made to be coincide with

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each other by shifting the phase as like fig. 5. This figure represents the spectra of TM 1 3 and TM 1 5 resulting from shifting the phase π/2. By comparing these spectra, it could be identified that the peak positions and the shapes are the same with each other. The amplitudes are not important factors in comparing the properties of these spectra. Usually, these values have been arbitrarily adopted for convenience of the schematic representation. So, it may be understood that these are the same spectra describing the eigen-modes of the rectangular wave guide filled with two different dielectrics. From these relation, it has been confirmed that FEM would be used to identify influence of the interface for more complex waveguide system.

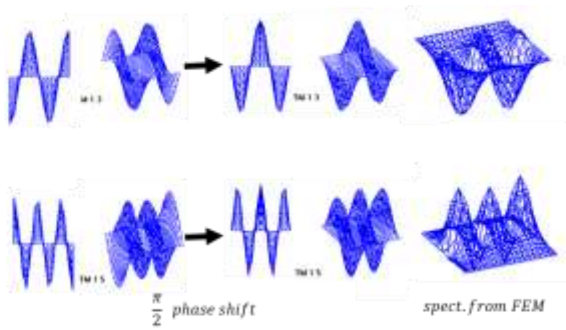

Fig. 5 Spectra resulted from the phase shift

CONCLUSION

To understand the eigen-property of the waveguie influenced by interface, the eigen equation was constructed using FEM. The Krylov-Schur iteration method has been applied to this equation to obtain the eigen-pairs. From the spectra, it could be identified that eigen-modes are dominently distributed arround the interface without differentiating TE or TM modes. These characteristics are distinguished from the results of the homogeneous waveguide. By comparing to the spectra based on the theoretical calculation, it would be confirmed that FEM may be applyed to understand the eigen-property of the waveguide influenced by the more complex interface.

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